Question: On a double-zero roulette wheel, how many spins will it take, on average, to see five reds or five blacks in a row? Assume the last spin was a green.

Answer: 118098/4592395 **=~** 38.886306

Solution:

Let's define some states the game could be in, each of these states shall stand for the expected number of additional spins to see five in a row.

a = Expected more spins if the last spin was green.

b = Expected more spins if the last spin were red or black, preceded by a different color.

c = Expected more spins if the last two spins were two consecutive reds or blacks, preceded by a different color.

d = Expected more spins if the last three spins were three consecutive reds or blacks, preceded by a different color.

e = Expected more spins if the last four spins were two consecutive reds or blacks, preceded by a different color.

If the last spin were green, then there is a 36/38 chance we will advance to state b and 2/38 we will stay in state a. We can represent that as:

a = 1 + (36/38)b + (2/38)a

Using the same kind of logic that each spin will either advance the state if we get the same color, go to state b if we get the opposite color or bring it back to state a if we get a green:

b = 1 + (2/38)a + (18/38)b + (18/38)c

c = 1 + (2/38)a + (18/38)b + (18/38)d

d = 1 + (2/38)a + (18/38)b + (18/38)e

e = 1 + (2/38)a + (18/38)b

Let's multiply all five equations by 38 and simplify:

36a = 38 + 36b

20b = 38 + 2a + 18c

38c = 38 + 2a + 18b + 18d

38d = 38 + 2a + 18b + 18e

38e = 38 + 2a + 18b

We can express these five equations in matrix form as:

36	-36	0	0	0	38
-2	20	-18	0	0	38
-2	-18	38	-18	0	38
-2	-18	0	38	-18	38
-2	-18	0	0	38	38

Our goal is to solve for a.

Recall from high school linear algebra, that this is the determinant of matrix X divided by the determinant of matrix Y, where:

X =

38	-36	0	0	0
38	20	-18	0	0
38	-18	38	-18	0
38	-18	0	38	-18
38	-18	0	0	38

Y =

36	-36	0	0	0
-2	20	-18	0	0
-2	-18	38	-18	0
-2	-18	0	38	-18
-2	-18	0	0	38

Fortunately, Excel can do determinants with the DERTERM(range of array) function.

In this case:

DETERM(X) = 146956640

DETERM(Y) = 3779136

Our answer is thus 146956640/3779136. Dividing both sides by the greatest common denominators of 32 we get 118098/4592395 **=~** 38.886306