Ultimate X Bonus Streak Analysis

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This paper extends an analysis of Ultimate X Video Poker to a new variation on its theme. Instead of an outcome generating an immediate return plus establishing a multiplier of the next round's return, in Bonus Streak a set of multipliers is established for subsequent hands. This paper analyzes this new type of game.

Key words: Gambling, non-discounted Markov Decision Problem, video poker, Ultimate X.

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1. Introduction

We refer the reader to our earlier paper analyzing Ultimate X[1] Poker [2] for basic concepts. Ultimate X Bonus Steak alters the basic idea of Ultimate X Poker by offering a stream of multipliers (a streak) for different outcomes to be applied to subsequent hands of play, not just a single multiplier for the next hand as in the original Ultimate X games. Like Ultimate X, this game costs twice the normal underlying game's maximum bet amount to activate the Bonus Streak (e.g., the normal maximal bet amount is 5 coins per line in Jacks or Better or Deuces Wild). That is, it cost 10 coins per line in Ultimate X. As is usual for multi-line games, each game starts with the same hand dealt to all lines of play and the held cards apply to each line. The outcomes come from independent draws from decks with the initial hand cards removed.

Table 1 shows per coin payouts (based on the initial 5 coins) and multiplier streaks for each possible outcome for a Deuces Wild game. For example, if on a line of play the current multiplier is 1 and one gets a Straight Flush then he will be paid 80 coins (5 times the outcome payout of 13). The "5" is because we are showing payouts on a per-coin bet basis and 5 coins were bet (the additional 5 coins wagered were to enable the bonus streak feature). This win sets up a streak so the next hand's multiplier will be 2, the subsequent 4 and so forth. However, if when in the midst of using a streak's multipliers, the player gets an outcome with another non-unit streak, then the current streak's remaining multipliers are changed to multipliers of 12.

Outcome	Per Coin Payout	Streak
Royal Straight Flush	**800**	**2,4,7,10,12**
Four Deuces	**200**	**2,4,7,10,12**
Wild Royal Straight Flush	**25**	**2,4,7,10,12**
Five of a Kind	**16**	**2,4,7,10,12**
Straight Flush	**13**	**2,4,7,10,12**
Four of a Kind (4K)	**4**	**2,2,4**
Full House (FH)	**3**	**2,2,4**
Flush	**2**	**2,2,4**
Straight	**2**	**1**
Three of a Kind (3K)	**1**	**1**
Nothing	**0**	**1**

Table 1: Ultimate X Bonus Steak Multiplies, Deuces Wild

[1] Both Ultimate X and Ultimate X Bonus Streak were created by IGT (https://www.igt.com/) and are offered in their video poker machines.

For example, suppose there is just one multiplier in place in the current streak for a line of play. Then let's track what happens with the following sequence of hands and outcomes shown in Table 2. The first hand results in a Three of a Kind and the payout is multiplied by the Outcome Multiplier of 1. The new streak is just "1". The Straight Flush with Hand 2 sets up a streak of future multipliers (2,4,7,10,12). We see these successively applied in the next two hands. However, the Full House outcome at Hand 4 would normally establish a streak of 2,2,4 but since we already have a streak longer than one element, the current remaining streak (7,10,12) is changed to all 12 multipliers (i.e., to 12,12,12).

Hand	Starting Streak	Outcome	Outcome Multiplier	New Streak
1	1	Three of Kind	1	1
2	1	Straight Flush	1	2,4,7,10,12
3	2,4,7,10,12	Nothing	2	4,7,10,12
4	4,7,10,12	Full House	4	12,12,12
5	12,12,12	Three of Kind	12	12,12
6	12,12	Nothing	12	12
7	12	Nothing	12	1
8	1	Nothing	1	1

Table 2: Example of Multiplier Evolution

Table 3 shows the possible streaks one might see at the start of a hand.

Streak	Streak Values
1	1
2	4
3	12
4	2,4
5	10,12
6	12,12
7	2,2,4
8	7,10,12
9	12,12,12
10	4,7,10,12
11	12,12,12,12
12	2,4,7,10,12

Table 3: Possible Observable Multiplier Streaks

2. Expected Value Analysis

Let M be the set of possible starting multiplier streaks. For example, for the streaks in Table 3 we have

$$M = \left\{ \begin{array}{l} (1),(2,2,4),(2,4),(4),(12,12),(12),(2,4,7,10,12), \\ (4,7,10,12),(7,10,12),(10,12),(12,12,12,12),(12,12,12) \end{array} \right\}.$$

Likewise, let Ω be the set of permutations of the elements of M taken L (the number of lines) at a time with repetition. So for a 3-Line game, each $\pi \in \Omega$ looks like $\pi = (\pi_1, \pi_2, \pi_3)$ where $\pi_i \in M$ and the j$^{\text{th}}$ multiplier of π_i is $\pi_i(j)$. Ω gives all of the possible streak states a player might see for the L lines before starting a hand of play.

Technically, the starting state of each round of play is (π, H) where $\pi \in \Omega$ results from the previous hands' outcomes and $H \in \mathbb{H}$ is a randomly generated next hand and $\mathbb{H}$ is the set of all possible starting hands. Since the outcome of any action depends on just (π, H) and what a decision maker chooses to hold in H, and not the history leading one to this state, the Markov property holds and the resulting problem is a Markov Decision problem[2]. This is not to say that all states can be reached in one step as was the case with the Ultimate X game in [2]. For example, for a one-line game, if the starting state is $((2,2,4), H)$, the only states that could be reached are $((2,4),*)$ and $((12,12),*)$. That is, the only realizable ending streaks are $(2,4)$ and $(12,12)$.

As in [2], we choose to study the non-discounted stream of returns and, for practical matters, assume the horizon is infinite. Thus we focus on solving the infinite horizon, non-discounted, Markov Decision problem (ndMDP) which is represented by

$$v_\pi + g = \sum_{H \in \mathbb{H}} P_H \max_i \left(\sum_{\ell=1}^{L} \pi_\ell(1) R_{H_i} + \sum_{\gamma \in \Omega} P_{\pi,\gamma}(H_i) v_\gamma \right) \qquad \pi \in \Omega$$
$$\sum_\pi P_\pi v_\pi = 0$$

[2] The associated Markov chains are readily shown to be ergodic.

Here g is the maximal gain per round of play, v_π is the relative bias for state $\pi \in \Omega$, P_π is the steady-state probability of being in state π (before a hand is dealt) under optimal decisions, and P_H is the probability of being dealt hand H. Note that $g/(2L)$ is the optimal expected return per bet unit for the game, the value we wish to compute. The "2" comes from the game costing twice the normal amount on which the payouts are based. For each hand, one must decide which of the possible $i = 1,\ldots,32$ ways to hold subsets of H, designated by H_i. Each possible decision results in an expected outcome for the hand, R_{H_i}, and a probability of transitioning to state γ of $P_{\pi,\gamma}(H_i)$.

Note that in the formulation above, we have reduced the starting state from (π, H) to π by averaging out the impact of the random starting hand (hence the $\sum_{H \in \mathbb{H}}$).

Since R_{H_i} is independent of the multipliers, let $m(\pi) = \sum_{\ell=1}^{L} \pi_\ell(1)$ and we can rewrite the problem as

$$
\begin{aligned}
&v_\pi + g = \sum_{H \in \mathbb{H}} P_H \max_i \left(m(\pi) R_{H_i} + \sum_{\gamma \in \Omega} P_{\pi,\gamma}(H_i) v_\gamma \right) \qquad \pi \in \Omega \\
&\sum_\pi P_\pi v_\pi = 0
\end{aligned}
\tag{1}
$$

Consider $P_{\pi,\gamma}(H_i)$. This is the probability of starting in state π and transitioning to state γ. This depends on which cards in H are held (designated by decision i leading to holding H_i) and the various possible outcomes (Straight, Flush, etc.) afterwards. Let $\mathbb{O}$ be the set of possible outcomes and $P_o(H_i \mid H)$ be the probability of outcome $o \in \mathbb{O}$ when cards H_i are held from hand H. For each outcome there is a payout and a streak (see Table 1 for example). The resulting streak is a function of the starting streak and the outcome represented by $s(o, \pi_\ell)$. Note, for regular Ultimate X, $s(o, \pi_\ell)$ is independent of π, it depends only on the hand's outcome. States in Bonus Streak having only single-length streaks also exhibit this property.

For example in a 2-Line game, if the starting state has $\pi = \left(\left(2,2,4\right), \left(1\right) \right)$ the possible resulting streaks are

$$\left(2,2,4\right) \rightarrow \begin{cases} \left(2,4\right) & o \in \left\{ Straight, 3K, Nothing \right\} \\ \left(12,12\right) & otherwise \end{cases}$$

$$\left(1\right) \rightarrow \begin{cases} \left(1\right) & o \in \left\{ Straight, 3K, Nothing \right\} \\ \left(2,2,4\right) & o \in \left\{ 4K, FH, Flush \right\} \\ \left(2,4,7,10,12\right) & otherwise \end{cases}$$

Thus

$$P_{\pi,\gamma_\ell}\left(H_i\right) \equiv \sum_{\substack{o \in \mathbb{O} \\ \gamma_\ell = s\left(o,\pi_\ell\right)}} P\left(o \mid H_i\right)$$

$$P_{\pi,\gamma}\left(H_i\right) = \prod_{\ell=1}^{L} P_{\pi,\gamma_\ell}\left(H_i\right) = \prod_{\ell=1}^{L} \sum_{\substack{o \in \mathbb{O} \\ \gamma_\ell = s\left(o,\pi_\ell\right)}} P\left(o \mid H_i\right)$$

and $P_{\pi,\gamma_\ell}\left(H_i\right)$ is the probability of outcomes having an associated multiplier of γ_ℓ given one starts in state $\left(\pi, H\right)$ and chooses to hold H_i. As in [2], we can iteratively solve (1) by

$$v_\pi^{n+1} + g^{n+1} = e_\pi^{n+1} = \sum_{H \in \mathbb{H}} P_H \max_i \left(m\left(\pi\right) R_{H_i} + \sum_{\gamma \in \Omega} P_{\pi,\gamma}\left(H_i\right) v_\gamma^n \right) \qquad \pi \in \Omega \tag{2}$$

$$g^{n+1} = \sum_{\pi} P_\pi^{n+1} e_\pi^{n+1}$$

$$P_\pi^{n+1} = \sum_{\gamma \in \Omega} P_\gamma^n P_{\gamma,\pi}\left(H_i^*\right) \qquad \pi \in \Omega$$

The term $P_{\gamma,\pi}\left(H_i^*\right)$ stands for the value of $P_{\pi,\gamma}\left(H_i\right)$ with an optimal decision i.

As discussed in [2], the number of permutations (with repetition) of $\left|M\right|$ things L at a time is $\left|M\right|^L$, so a 10-Line version of Ultimate X Bonus Streak with the multipliers shown in Table1 has $12^{10} = 61{,}917{,}364{,}224$ multiplier patterns a player may see. So the true number of states is $\binom{n}{5}\left|M\right|^L$ where n is the size of the deck of cards used (assuming order of the cards is not important). For example, for decks of 52 cards and a 10-line game, the number of states is on the order of 10^{17}, over 100 quadrillion.

Fortunately, some of the problem size reductions discussed in [2] can be used in the Bonus Streak game. In particular, the reductions are:

1. Use equivalent suite permutations of hands to reduce $H \in \mathbb{H}$ to unique hands $H \in \overline{\overline{\mathbb{H}}}$. This is easily implemented by letting P_H reflect the number of different suite permutations for a given hand. For games with 52 cards, this reduces the size of $\mathbb{H}$ from 2,598,960 hands to 134,459 in $\overline{\overline{\mathbb{H}}}$.
2. Use state permutations to reduce the state space. For example, in a 3-Line game, state $\{(1),(2,4),(12,12)\}$ will give the same expected payouts as state $\{(2,4),(1),(12,12)\}$ and state $\{(1),(12,12),(2,4)\}$ since the order of the multipliers across the lines of play is not important. As in [2] we let $C \subseteq \Omega$ contain just the unique combinations (say those in sorted order) and denote equivalent states $\gamma \approx \pi$ in Ω for each $\gamma \in C$.

Unfortunately, a third reduction in [2] first suggested by Michael Shackelford [4] is not valid here. That reduction stated that all states having the same value of $m(\pi)$ are equivalent. The proof given in [2] relied on the fact that $P_{\pi,\gamma_\ell}(H_i)$ was independent of π which is not the case with Bonus Streak unless the states are composed of single-length streaks.

Let $C \subseteq \Omega$ contain just the unique combinations (say those in sorted order). So

$$|C| = \binom{|M|+L-1}{|M|-1}.$$

With the reductions, we wish to solve

$$v_\pi^{n+1} + g^{n+1} = e_\pi^{n+1} = \sum_{H \in \overline{\overline{\mathbb{H}}}} P_H \max_i \left(m(\pi) R_{H_i} + \sum_{\gamma \in C} P_{\pi,\gamma}(H_i) v_\gamma^n \right) \qquad \pi \in C \tag{3}$$

$$g^{n+1} = \sum_\pi P_\pi^{n+1} e_\pi^{n+1}$$

$$P_\pi^{n+1} = \sum_{\gamma \in \Omega} P_\gamma^n P_{\gamma,\pi}\left(H_{i \in S_{\gamma,H}}\right) \qquad \pi \in \Omega$$

With the reductions, we need to adjust our definition of $P_{\pi,\gamma}\left(H_i\right)$. Let

$$P_{\pi,\gamma}\left(H_i\right)=\prod_{\ell=1}^{L}\sum_{\substack{\eta\in\Omega\\ \eta\approx\gamma}}P_{\pi,\eta_\ell}\left(H_i\right)=\prod_{\ell=1}^{L}\sum_{\substack{\eta\in\Omega\\ \eta\approx\gamma}}\sum_{\substack{o\in\mathbb{O}\\ \eta_\ell=s(o,\pi_\ell)}}P\left(o\,|\,H_i\right)\qquad \pi,\gamma\in\Omega$$

Note, the original values are $v_\gamma^{n+1}=v_\pi^{n+1}$ for $\gamma\in\Omega/C,\gamma\approx\pi$. As in [2], we stop (3) when

$$\left|g^{n+1}-g^{n}\right|+\sum_{\pi\in C}\left|v_\pi^{n+1}-v_\pi^{n}\right|+\sum_{\pi\in C}\left|P_\pi^{n+1}-P_\pi^{n}\right|<10^{-10}\left|C\right|. \tag{4}$$

We solved a hypothetical[3] 1-Line version of the Deuces Wild game in Table 1 to get a gain (g) of 1.94665 and steady state values shown in Table 4. The Expected Value (EV) is 1.94665/2 = 0.973325.

Deuces Wild – 1 Line		
π	v_π	P_π
1	-2.506	0.680794
4	0.363605	0.069155
12	8.08679	0.031268
2,4	1.30152	0.079446
10,12	15.8166	0.006048
12,12	17.7519	0.014929
2,2,4	3.38067	0.091454
7,10,12	20.8671	0.006858
12,12,12	27.4171	0.002111
4,7,10,12	23.5686	0.007795
12,12,12,12	37.0822	0.001174
2,4,7,10,12	25.2629	0.008969

Table 4: Solution to one line version of the game with multiples in Table 2

Table 5 gives the outcomes for the 1-3 Line versions of this Deuces Wild game. Actual machines in casinos currently only offer 3, 5 and 10-Line versions, so the 1-Line and 2-Line versions are hypothetical.

[3] Although we have not seen a 1-Line version of the game, we anticipate their introduction just as 1-Line games of Ultimate X were eventually released by IGT.

Deuces Wild Video Poker	g	EV
1-Line	1.94665	0.973325
2-Lines	3.88404	0.971010
3-Lines	5.81832	0.969721

Table 5: Optimal expected returns for Deuces Wild Ultimate X Bonus Streak.

Interestingly, the Bonus Streak game appears to exhibit the same phenomenon that the Ultimate X games showed (Page 16, [2]):

> “the impact on expected return as the number of lines increases is negative“

Note the EVs reduce as the number of lines increase in Table 5.

As another example, Table 6 gives the payouts and streaks for 7-5 Bonus Poker Deluxe.

Outcome	Payout	Streak
Royal Straight Flush	800	2,5,8,10,12
Straight Flush	50	2,5,8,10,12
Four of a Kind (4K)	80	2,5,8,10,12
Full House (FH)	7	2,5,8,10,12
Flush	5	2,5,8
Straight	4	2,5
Three of a Kind (3K)	3	2,5
Two Pair	1	1
Jacks or Better Pair	1	1
Nothing	0	1

Table 6: Ultimate X Bonus Steak Multiplies, Bonus Poker Deluxe

Table 7 gives the outcomes for the 1-3 Line versions of Bonus Poker Deluxe and Table 8 its steady-state values for 1-Line.

Bonus Deluxe	g	EV
1-Line	1.93818	0.969092
2-Lines	3.86879	0.967198
3-Lines	5.79847	0.966412

Table 7: Optimal expected returns for Bonus Poker Deluxe Ultimate X Bonus Streak.

Bonus Deluxe – 1 Line		
π	v_π	P_π
1	-2.43554	0.750595
5	1.3749	0.064623
8	4.2542	0.011538
12	8.10293	0.023618
2,5	2.14009	0.073308
5,8	7.56512	0.013025
10,12	15.79	0.008086
12,12	17.7151	0.00536
2,5,8	8.7366	0.014784
8,10,12	21.7695	0.009117
12,12,12	27.3272	0.002571
5,8,10,12	25.2758	0.010296
12,12,12,12	36.9393	0.001391
2,5,8,10,12	26.6274	0.011688

Table 8: Optimal relative biases and steady state probabilities for Bonus Deluxe.

The challenge with analyzing games beyond 3-Lines is easily seen in Table 9 where we show the sizes of the states for the Deuces Wild game of Table 1.

	1-Line	3-Lines	5-Lines	10-Lines
$\lvert\Omega\rvert = \lvert M\rvert^L$	12	1,728	248,832	61,917,364,224
$\lvert C\rvert = \binom{\lvert M\rvert + L - 1}{L}$	12	364	4,368	352,716

Table 9: Size of Sets for Ultimate X Bonus Streak Deuces Wild

For example, even using the state reduction to C for a 10-Line game, results in 352,715 states. For each state we need to find the optimal hold of 134,459 hands, each requiring 32 probability vectors and expected value calculations. That is, over 1.5 trillion calculations for each are needed at each iteration in (3). With Ultimate X, the third state size reduction (which is not generally applicable here) to set D (in [2]) reduced the state space size dramatically. For the Deuces Wild game examined in [2], the sizes are as shown in Table 10. Notice that the 10-Line Ultimate X game was easier to solve than the 3-Line game of Bonus Streak Ultimate X.

	1-Line	3-Lines	5-Lines	10-Lines
$\lvert\Omega\rvert = \lvert M\rvert^L$	7	343	16,807	282,475,249
$\lvert C\rvert = \binom{\lvert M\rvert + L - 1}{L}$	7	84	462	8,008
$\lvert D\rvert$	7	29	51	106

Table 10: Size of Sets in [2] for Ultimate X Deuces Wild

In short, without some massively parallel computing platform, some new insights are needed to solve the Bonus Streak versions of Ultimate X for 10-Line games. 5-Line games are within reach but will take weeks to solve.

3. Possible Speed-ups

Some obvious computational speed-ups include precomputing the following values which don't change from iteration to iteration:

1. $P_H m(\pi) R_{H_i} \qquad \forall H \in \overline{\mathbb{H}}, i = 1, \ldots, 32$
2. $P_H P_{\pi,\gamma}(H_i) \qquad \forall H \in \overline{\mathbb{H}}, \pi, \gamma \in C, i = 1, \ldots, 32$

The second suggestion above may be impractical because C grows so fast and $\overline{\mathbb{H}}$ is large.

Similarly, dividing the iterations to parallel computations over $\overline{\mathbb{H}}$ and C are easily done. With most processors implementing multiple cores and hyper-threading, parallel computing is possible[4].

As mentioned when discussing state-space reductions, it was noted we can have a small reduction of states by collapsing those states having all single-length streaks and equal $m(\pi)$ values. The impact is minimal, however. For example, in the Jacks or Better game shown in Section 4 below, the 3-Line game has 560 states in C and only 17 can be reduced using this

[4] We used 10 of our 12 cores on a Xeon E5645 Intel processor.

equivalence. The overhead to implement this reduction hardly covers the slight reduction in state space size.

Another possible speed-up can be achieved using a termination criterion first suggested by Odoni [3]. He showed that

$$\overline{L}^n \geq \overline{L}^{n+1} \geq g \geq \underline{L}^{n+1} \geq \underline{L}^n$$

$$\overline{L}^n = \max_{\pi} e_{\pi}^{n+1} - v_{\pi}^n$$

$$\underline{L}^n = \min_{\pi} e_{\pi}^{n+1} - v_{\pi}^n$$

So, stopping when $\overline{L}^{n+1} - \underline{L}^{n+1} < \varepsilon$ will provide a good estimate of g for small enough ε. For examples, for the first Jacks or better game shown later using ε values shown in the Table below, we found the following number of iterations needed to achieve the stopping condition:

Lines	**1**	**2**	**3**
Iterations with Condition (4)	**28**	**29**	**30**
Iterations with $\varepsilon = 10^{-8}$	**25**	**25**	**26**
Iterations with $\varepsilon = 10^{-7}$	**22**	**24**	**24**
Iterations with $\varepsilon = 10^{-6}$	**21**	**21**	**21**

This stopping criterion may not leave us with as accurate estimates of the steady state probabilities or relative bias values as the stopping criterion discussed earlier with Equation (4), but it could save iteration rounds if we are interested in just computing the gain of a game.

In [2] we discussed some additional computational reductions. One was to use other forms of iteration where both storage requirements and rate of convergence improved when applicable. Such methods exist for solving discounted, infinite-horizon, Markov Decision problems. However, we know of no way to implement these for the non-discounted problem without first converting it to a form where they can be applied (as done by Koehler et al. in [1]) which itself required solving a Markov decision problem.

We also mentioned it is possible to permanently eliminate sub-optimal decisions as the iteration proceeds, thus, in principle, reducing the problem size. In our explorations of this approach, the overhead introduced did not justify the improvement in convergence speed.

4. Results

Below are the results we found for a selection of games, pay tables and bonus streaks for 1-Line and 3-Line versions of the game.

Outcome	Payout	Streak	Payout	Streak
Royal Straight Flush	**800**	**2,5,8,10,12**	**800**	**2,5,8,10,12**
Straight Flush	**50**	**2,5,8,10,12**	**50**	**2,5,8,10,12**
Four of a Kind	**25**	**2,5,8,10,12**	**25**	**2,5,8,10,12**
Full House	**8**	**2,5,8,10,12**	**8**	**2,5,8,10,12**
Flush	**6**	**2,5,8**	**5**	**2,5,8**
Straight	**4**	**2,5**	**4**	**2,5**
Three of a Kind	**3**	**2,5**	**3**	**2,5**
Two Pair	**2**	**1**	**2**	**1**
Jacks or Better Pair	**1**	**1**	**1**	**1**
Nothing	**0**	**1**	**0**	**1**
EV of Regular Game	**0.983927**		**0.972984**	
1-Line Bonus Streak EV	**0.995064**		**0.980650**	
3-Line Bonus Streak EV	**0.992695**		**0.977559**	

Jacks or Better

Outcome	Payout	Streak
Royal Straight Flush	**800**	**2,4,8,10,12**
Straight Flush	**50**	**2,4,8,10,12**
Four Aces	**80**	**2,4,8,10,12**
Four 2s-4s	**40**	**2,4,8,10,12**
Four 5s-Ks	**25**	**2,4,8,10,12**
Full House	**7**	**2,4,8,10,12**
Flush	**5**	**2,4,8**
Straight	**4**	**2,4**
Three of a Kind	**3**	**2,4**
Two Pair	**2**	**1**
Jacks or Better Pair	**1**	**1**
Nothing	**0**	**1**
EV of Regular Game	**0.980147**	
1-Line Bonus Streak EV	**0.987757**	
3-Line Bonus Streak EV	**0.984631**	

Bonus Poker

Outcome	Payout	Streak
Royal Straight Flush	**800**	**2,5,8,10,12**
Straight Flush	**50**	**2,5,8,10,12**
Four of a Kind	**80**	**2,5,8,10,12**
Full House	**7**	**2,5,8,10,12**
Flush	**5**	**2,5,8**
Straight	**4**	**2,5**
Three of a Kind	**3**	**2,5**
Two Pair	**1**	**1**
Jacks or Better Pair	**1**	**1**
Nothing	**0**	**1**
EV of Regular Game	**0.962526**	
1-Line Bonus Streak EV	**0.969092**	
3-Line Bonus Streak EV	**0.966412**	

Bonus Poker Deluxe

Outcome	Payout	Streak
Royal Straight Flush	**800**	**2,4,8,10,12**
Straight Flush	**50**	**2,4,8,10,12**
Four Aces	**160**	**2,4,8,10,12**
Four 2s-4s	**80**	**2,4,8,10,12**
Four 5s-Ks	**50**	**2,4,8,10,12**
Full House	**9**	**2,4,8,10,12**
Flush	**5**	**2,4,8**
Straight	**4**	**2,4,8**
Three of a Kind	**3**	**2,4**
Two Pair	**1**	**1**
Jacks or Better Pair	**1**	**1**
Nothing	**0**	**1**
EV of Regular Game	**0.952738**	
1-Line Bonus Streak EV	**0.962197**	
3-Line Bonus Streak EV	**0.959335**	

Double Bonus Poker

Outcome	Payout	Streak
Royal Straight Flush	**800**	**2,4,8,10,12**
Straight Flush	**50**	**2,4,8,10,12**
Four Aces w 234	**400**	**2,4,8,10,12**
Four 2s-4s w A-4	**160**	**2,4,8,10,12**
Four Aces 5s-Ks	**160**	**2,4,8,10,12**
Four 234 w 5s-Ks	**80**	**2,4,8,10,12**
Four 5s-Ks	**50**	**2,4,8,10,12**
Full House	**9**	**2,4,8,10,12**
Flush	**5**	**2,4,8**
Straight	**4**	**2,4,8**
Three of a Kind	**3**	**2,4**
Two Pair	**1**	**1**
Jacks or Better Pair	**1**	**1**
Nothing	**0**	**1**
EV of Regular Game	**0.978729**	
1-Line Bonus Streak EV	**0.991220**	
3-Line Bonus Streak EV	**0.987923**	

Double Double Bonus Poker

Outcome	Payout	Streak
Royal Straight Flush	**800**	**2,4,8,10,12**
Straight Flush	**50**	**2,4,8,10,12**
Four Aces w 234	**800**	**2,4,8,10,12**
Four 2s-4s w A-4	**400**	**2,4,8,10,12**
Four Aces 5s-Ks	**160**	**2,4,8,10,12**
Four 234 w 5s-Ks	**80**	**2,4,8,10,12**
Four 5s-Ks	**50**	**2,4,8,10,12**
Full House	**9**	**2,4,8,10,12**
Flush	**6**	**2,4,8**
Straight	**5**	**2,4,8**
Three of a Kind	**2**	**2,4**
Two Pair	**1**	**1**
Jacks or Better Pair	**1**	**1**
Nothing	**0**	**1**
EV of Regular Game	**0.981540**	
1-Line Bonus Streak EV	**0.993189**	
3-Line Bonus Streak EV	**0.990460**	

Triple Double Bonus Poker

Outcome	Per Coin	Streak
Royal Straight Flush	**800**	**2,4,7,10,12**
Four Deuces	**200**	**2,4,7,10,12**
Wild Royal Straight	**25**	**2,4,7,10,12**
Five of a Kind	**16**	**2,4,7,10,12**
Straight Flush	**13**	**2,4,7,10,12**
Four of a Kind (4K)	**4**	**2,2,4**
Full House (FH)	**3**	**2,2,4**
Flush	**2**	**2,2,4**
Straight	**2**	**1**
Three of a Kind (3K)	**1**	**1**
Nothing	**0**	**1**
EV of Regular Game	**0.967651**	
1-Line Bonus Streak EV	**0.973327**	
3-Line Bonus Streak EV	**0.969721**	

Deuces Wild

Outcome	Per Coin	Streak
Royal Straight Flush	**800**	**2,4,6,10,12**
Four Deuces w Ace	**400**	**2,4,6,10,12**
Four Deuces	**200**	**2,4,6,10,12**
Wild Royal Straight	**25**	**2,4,6,10,12**
Five Aces	**80**	**2,4,6,10,12**
Five 3’s-5’s	**40**	**2,4,6,10,12**
Five 6’s-K’s	**20**	**2,4,6,10,12**
Straight Flush	**10**	**2,4,6,10,12**
Four of a Kind (4K)	**4**	**2,2,4**
Full House (FH)	**3**	**2,2,4**
Flush	**3**	**2,2,4**
Straight	**1**	**1**
Three of a Kind (3K)	**1**	**1**
Nothing	**0**	**1**
EV of Regular Game	**0.973644**	
1-Line Bonus Streak EV	**0.989868**	
3-Line Bonus Streak EV	**0.986977**	

Bonus Deuces Wild

5. Summary

This paper presented an analysis of Ultimate X Bonus Streak games. This generalizes the results of Ultimate X games [2] since Ultimate X can be considered as a special case of Ultimate X Bonus Streak. However, Ultimate X can be solved faster using reductions that can't be used with Bonus Streak games.

At the present time, we are unable to solve Bonus Streak games with 10-Lines because the state space grows too fast. 5-Line games are within reach, but we have not solved them yet. We are working on new insights and algorithmic improvements.

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